

Beyond pixel-wise supervision:

Semantic segmentation with higher-order shape descriptors

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Motivation











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Contributions

• Re-introduce shape descriptors, in the context of deep learning-based semantic segmentation;

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- Re-introduce shape descriptors, in the context of deep learning-based semantic segmentation;
- propose a way to supervise using *only* shape descriptors.

Ground truth



Trained with cross-entropy 65536 discrete labels



Trained with shape descriptors 4 descriptors per class



Computing shape descriptors



$$\mu_{\mathrm{p},\mathrm{q}}^{(\mathrm{k})}(\mathrm{s}_{\boldsymbol{\theta}}) := \sum_{\mathrm{i}\in\Omega} \mathrm{s}_{\boldsymbol{\theta}}^{(\mathrm{i},\mathrm{k})} \mathrm{x}_{(\mathrm{i})}^{\mathrm{p}} \mathrm{y}_{(\mathrm{i})}^{\mathrm{q}},$$



General shape moment:

$$\mu_{\mathbf{p},\mathbf{q}}^{(k)}(s_{\boldsymbol{\theta}}) := \sum_{i\in\Omega} s_{\boldsymbol{\theta}}^{(i,k)} x_{(i)}^{\mathbf{p}} y_{(i)}^{\mathbf{q}},$$

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Centroid: "average" of pixels coordinates

$$\mathfrak{C}^{(\mathrm{k})}(\mathrm{s}_{\boldsymbol{\theta}}) := \left(\frac{\mu_{1,0}^{(\mathrm{k})}(\mathrm{s}_{\boldsymbol{\theta}})}{\mu_{0,0}^{(\mathrm{k})}(\mathrm{s}_{\boldsymbol{\theta}})}, \frac{\mu_{0,1}^{(\mathrm{k})}(\mathrm{s}_{\boldsymbol{\theta}})}{\mu_{0,0}^{(\mathrm{k})}(\mathrm{s}_{\boldsymbol{\theta}})}\right)$$

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Central moment

General central moment:

$$\bar{\mu}_{p,q}^{(k)} := \sum_{i \in \Omega} s_{\theta}^{(i,k)} \left(x_{(i)} - \frac{\mu_{1,0}^{(k)}}{\mu_{0,0}^{(k)}} \right)^{p} \left(y_{(i)} - \frac{\mu_{0,1}^{(k)}}{\mu_{0,0}^{(k)}} \right)^{q},$$

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Average distance to the centroid: "standard deviation" of pixels coordinates

$$\mathfrak{D}^{(\mathrm{k})}(\mathrm{s}_{\boldsymbol{\theta}}) := \left(\sqrt[2]{\frac{\bar{\mu}_{2,0}^{(\mathrm{k})}(\mathrm{s}_{\boldsymbol{\theta}})}{\mu_{0,0}^{(\mathrm{k})}(\mathrm{s}_{\boldsymbol{\theta}})}}, \sqrt[2]{\frac{\bar{\mu}_{0,2}^{(\mathrm{k})}(\mathrm{s}_{\boldsymbol{\theta}})}{\mu_{0,0}^{(\mathrm{k})}(\mathrm{s}_{\boldsymbol{\theta}})}}\right).$$

Other descriptors

Length: "perimeter" of the prediction

$$\mathfrak{L}^{(k)}(s_{\boldsymbol{\theta}}) := \sum_{i,j \in \mathcal{G}_{\Omega}} |s_{\boldsymbol{\theta}}^{(i,k)} - s_{\boldsymbol{\theta}}^{(j,k)}|,$$

 $i,j\in \mathcal{G}_\Omega \quad \text{ pairs of neighbor pixels.}$

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Ratio of descriptors:

$$\mathfrak{R}_{\mathfrak{f}}^{(k,l)}(s_{\boldsymbol{\theta}}) := \frac{\mathfrak{f}^{(k)}(s_{\boldsymbol{\theta}})}{\mathfrak{f}^{(l)}(s_{\boldsymbol{\theta}})}$$

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 $\mathfrak{f} \in \{\mathfrak{V}, \mathfrak{C}, \mathfrak{D}, \mathfrak{L}\}$

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Example:



The myocardium (green) surrounds the left-ventricle (yellow):

$$\frac{\mathfrak{L}^{(\text{green})}}{\mathfrak{L}^{(\text{yellow})}} \geq 2$$

Series of losses optimized with SGD:

$$\underset{\boldsymbol{\theta}}{\operatorname{arg\,min}} \quad \sum_{\mathfrak{f} \in \{\mathfrak{V}, \mathfrak{C}, \mathfrak{D}, \mathfrak{L}\}} \sum_{k} \left[\widetilde{\psi}_{t} \left(0.9 \tau_{\mathfrak{f}}^{(k)} - \mathfrak{f}^{(k)}(s_{\boldsymbol{\theta}}) \right) + \widetilde{\psi}_{t} \left(\mathfrak{f}^{(k)}(s_{\boldsymbol{\theta}}) - 1.1 \tau_{\mathfrak{f}}^{(k)} \right) \right],$$

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 $\tau_{\mathfrak{f}}$: ground-truth descriptor, $\mathfrak{f}(s_{\theta})$: descriptor on prediction, $\widetilde{\psi}_{t}(z)$: extended log-barrier:



Allows us to enforce $0.9\tau_{\mathfrak{f}} \leq \mathfrak{f}(s_{\theta}) \leq 1.1\tau_{\mathfrak{f}}$.

Experiments

Test on two datasets: ACDC (heart segmentation, 4 classes) and PROMISE12 (prostate, 2 classes).

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Results



Metrics numbers available in the paper.

Future works: generalization to 3D and time series

Descriptors as bounds can be valid for multiple time-points:



Ellipsoids for each class at two time points: systole and diastole (ACDC dataset).

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code available online: https://github.com/HKervadec/shape_descriptors.